DIAGNOSIS OF WEAKNESSES AND REMEDIAL TEACHING IN CLASS VI

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I-INTRODUCTION

II-DIAGNOSIS OF WEAKNESSES

  Informal Procedures for Diagnosis of Weaknesses

  Diagnostic Tests

  Sequencing of Objectives

  Selection of Objectives

III-REMEDIAL TEACHING

  Causes of Weaknesses

  Decisions about Remedial Teaching

  Guidelines for Remedial Teaching

  Specific Suggestions for Remedial Teaching in class VI

IV-CONCLUDING COMMENTS

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I-INTRODUCTION

The achievement of students in mathematics in most schools by any criterion-internal evaluation, teachers' observations, external examination and research and evaluation studies is low and there is a need to improve it. An important reason for this is the lack of mastery of fundamentals (Gupta, 1971, Rastogi, 1991). Mathematics learning largely consists of building understanding of new concepts onto previously understood concepts. Learning of a very simple procedure like addition of two digit numbers with renaming calls for mastery of several prerequisites such as reading and writing of two or three digit numbers and symbol of addition, concept of addition, basic facts of addition (addition of one digit numbers), concept of place value-renaming a sum of ones greater than nine as tens and ones and adding these tens to the sum of tens. A pupil lacking any of these would not be able to add numbers correctly and learn other procedures or apply and solve problems which require it as a component skill. There is thus a need to assure mastery of such prerequisites by giving a mastery test after teaching these and checking it before teaching topics for which these are prerequisites and doing remedial teaching if necessary.

In this monograph we discuss the diagnosis of weaknesses in whole numbers and remedial teaching of those while teaching whole numbers at the beginning of class VI. As NCERT syllabi and/or textbooks are widely used all over India, we would use that and situation in government school in Delhi with which we are familiar as reference points.

Adequate mastery of whole numbers and operations on them is a prerequisite for learning of other number systems, commercial mathematics, measurement and algebra at upper primary stage. The students are expected to have attained mastery of whole numbers by the end of class V per National Council of Educational Research and Training syllabi for primary stage (NCERT, 2005). As class VI syllabus only discusses large numbers and properties of whole number. Topics like HCF, LCM, measurement, ratio and proportion require that students can carry out the procedures accurately and quickly. Most of the government schools in Delhi begin with class VI, follow NCERT syllabus and/or textbooks and use Hindi as a medium of instruction, have trained teachers with mathematics at least up to B.A./B.Sc. and relatively better physical facilities than other schools in the country. Teaching up to class V is imparted in attached primary schools run by Municipal Corporation of Delhi. These schools share the characteristics of secondary schools to a lesser degree and all subjects including mathematics are taught by the same teacher who may have had mathematics up to class X only.

A crucial problem identified by teachers in teaching class VI students is the lack of fundamentals-reading and writing of numbers (in Hindi), automaticity (answering without thinking) of basic facts -addition and multiplication of one digit numbers and their subtraction and division corollaries and execution of operations other than addition of multi-digit numbers and adequate understanding of concepts and procedures. This was confirmed by formal testing of students in some government schools by us (Mahajan, 1995, Institute of Advanced Studies in Education, 1997). There is thus a need to identify weaknesses in whole numbers and do remedial teaching for those. The task is not as difficult as teachers seem to feel. There is research evidence to show that devoting a little time to it can considerably improve performance in basic skills as well as overall mathematics achievement (Gupta, 1971; Rastogi, 1991).

There may be many reasons for weaknesses in mathematics such as physical disability, emotional disturbance, poor home environment, poor facilities at home for studying, need to earn or help parents in their vocation after school, low aptitude, low motivation and deficiencies in the curriculum. As most students in government schools especially those that draw children from lower socioeconomic stratum of the society have failed to learn much, poor home environment and curricular deficiencies seem to be the predominant causes. As schools cannot do much about improving home environment, except keep it in view, we will discuss curricular deficiencies that may be the causes of weaknesses.

A curriculum is an operational plan for instruction that specifies short and long term objectives-what the student should be able to do at the end of a lesson, unit, grade or stage of education; teaching-learning activities that would help in achieving these and evaluation of these objectives with the aim of enhancing these. NCERT has published syllabi which provide a vision of teaching and syllabi for different classes in mathematics. These have also been operationalized by NCERT into textbooks. The teachers generally do not have access to the syllabus. Textbook is the curriculum for most teachers and at best they try to teach what is given in the textbook. Teaching by and large consists of verbal explanations with some examples and activities for learning usually consist of doing textbook exercises. Evaluation mainly seeks evidence on facts, and computational skills and contrived applications done earlier as class or home work and is confined to some tests and two mid-term and an annual examination for grading purposes.

An examination of the syllabi and textbooks for classes I to V (NCERT, 2005) suggest the following deficiencies:

- There is little emphasis on automaticity of basic facts -addition and multiplication of one digit numbers and their subtraction and division corollaries. Practice provided for execution of fundamental operations is not sufficient for mastery of these leave alone fluency which is necessary for problem solving, applications and further learning.
- Neglect of manipulatives for understanding of concepts important at this stage of learning viz. the stage of concrete operations, more so for lower class children. It mostly depends on stories with pictures with applications to daily life for understanding and integration of concepts which many children especially from lower class may find difficult due to poor language and computation skills.
- NCERT (2005) books for primary classes do make mathematics fun by having pictures, stories, games and applications to daily life. But they have neglected the systematic mastery of the fundamental building blocks necessary for success in applications, problem solving and further learning. It has not only radically diminished the content in numbers and paper and pencil work and shunned things like algorithms and repeated practice. The assumption that if we present school mathematics through pictures, stories, and games and emphasize understanding and applications to daily life, the students will be motivated to learn more mathematics and will somehow learn the mathematics needed for further learning, applications and problems solving is not correct. The students must know the mathematics before they can apply it. There is a need to emphasize mastery of basic facts and mastery and fluency in fundamental operations of desired complexity with understanding. The applications of these to daily life and mathematics then should require use of fundamental operations of that complexity only unless calculators are available for use by students. The need for this understanding is not obviated by the availability of calculators and computers as the experience of implementation of NCTM standards in nineties in USA show. The standards by NCTM (1989) had also deemphasized computations but had to revert back to emphasizing those. Joan Ferrini-Mundy, chairwoman of the committee to revise the standards releasing the revised standards at a news conference during a convention attended by 18,000 mathematics teachers, said that students must be fluent in arithmetic computation, use efficient and accurate methods, and understand those methods. They should know their basic addition, subtraction, multiplication and division combinations as well as understand them.
- Prerequisites have been ignored in sequencing content. For example, questions on count and match (pp.25-31) are given before numbers are introduced (pp 31-42). Applications require addition and subtraction of

five-digit numbers (p.14 in class V) when the syllabus is confined to three-digit numbers.

- No practice questions are given on some important skills e.g. in class 2, the number names, comparison of numbers, dictation of number, translation from words to figures and vice versa and place value of numbers and are missing except for four questions only. Without discussing counting back or forward and what a century is, questions on how many more are needed to make one or triple century.
  It is assumed that all children can generalize easily from a few examples of two-digit numbers on how to write all two-digit numbers or three-digits and four-digit numbers.
- It does not prepare children for middle school math that requires accurate and speedy computations. The research on precision teaching (that emphasizes accuracy as well as speed) shows that it enhances achievement considerably.

As teachers mainly depend on the textbook, these deficiencies are likely to be reflected in teaching also. More so as all subjects up to class V are taught by the same teacher. Most of these teachers do not have good understanding of mathematical concepts and procedures themselves nor have access to teaching guides or books on teaching of mathematics (findings of survey on teachers). They are not likely to use manipulatives on their own due to lack of their knowledge, availability or physical facilities for their use or make up for other deficiencies.

Students' performance in achievement surveys (NCERT, 1994) and research studies indicate that teachers do not provide for and assure mastery of minimum essentials necessary for further learning. (A survey was conducted in several districts under the district primary education programme (DPEP, 1994) on achievement in mathematics. The sample consisted of 23056 students belonging to 106 blocks, 91 urban areas in 46 districts in Assam, Haryana, Madhya Pradesh, Maharashtra, Orissa, Kerala and Karnataka. It revealed that in class II only 26.5% in Haryana, 31.1% in Madhya Pradesh, 36.6% in Orissa, 34.7% in Assam and 16.7% in Maharashtra attained mastery (80% or above) in recognition of numbers. Similarly 34.8% in Haryana, 28% in Madhya Pradesh, 38.4% in Orissa 30.8% in Assam and 19.9% in Maharashtra in class III attained mastery in addition and subtraction.

All India Annual Status Education Report (ASER) 2008 (http://www.asercentre.org) was conducted in 578 out of 583 districts in rural India by randomly selecting 30 villages per district using the village directory of the 2001 census. The sampling was done using the PPS (Probability Proportional to Size Sampling) technique. This method allows

villages with larger populations to have a higher chance of being selected in the sample. Twenty households were selected in each village and all the children in the age-group 3-16 in the household were tested individually. In mathematics the student were asked to recognize number from 0-99 subtract two-digit numbers requiring renaming (that should have been mastered by class II), divide a three-digit number by one-digit number (that should have been mastered by class IV).
Percentage of children in classes I-V who could perform these was as follows

Grade	Does not know any thing	Recognize numbers 1-9	Recognize numbers 10-99	Subtract two-digit numbers	Divide a three-digit number by one-digit number
I	34.6	65.4	22	4.4	1.3
II	12.6	87.4	52	16.7	3.9
III	5.6	94.4	73.1	38.7	10.8
IV	2.8	97.2	84.6	57.3	24
V	1.8	98.2	90.4	67.8	37

The children were also asked to tell time at 15-minute intervals and 5-minute intervals, the percentage of children in Grades I-V who could do these was as follows:

Grade	Could tell time on a clock at 5-minute and 15-minute intervals	Could compare money involving 2-4 five-rupee notes and 1-2 ten-rupee notes and could add 3 numbers involving 10, 50 and 100
I	7	21.4
II	17.5	42.6

III	32.9	61.9
IV	47.7	75.1
V	60.9	83.2

This makes further learning increasingly difficult which is reflected in low performance in topics requiring these as prerequisites and higher classes. Lack of continuous evaluation for assuring mastery and guiding teaching-learning is another important reason. The evaluation is mainly used for grading purposes only as required by the school or directorate.

We need to keep these deficiencies in mind for diagnosis of weaknesses and planning of remedial teaching.

Remedial teaching can be organized at several levels-by the department for a block of schools, by a school for children that need it in all sections or in each section of class VI and can be conducted at several levels e.g. individual, small group or large group depending on the number of students that need it and resources. As large number of students that need it and limited resources rule out individual or small group teaching or individual learning by students on their own under teachers supervision from teaching-learning materials due to lack of suitable materials and adequate reading skills. We will confine ourselves to remedial teaching that may be conducted by directorate, schools or teacher to large groups with emphasis on understanding to take care of misconceptions which vary from student to student and provision for individual differences in minimum essentials to provide for their mastery.

II-DIAGNOSIS OF WEAKNESSES

A prerequisite for planning remedial teaching is to identify students that need remedial teaching, areas in which they need it and within each area the kind of retraining needs.

Readiness for Learning in Class VI

In class VI except for numbers larger than 1000, whole numbers are discussed as a system for extending the system to integers, rational numbers and algebra. It is assumed by class V children have mastered the fundamental operations for further learning for which these are prerequisites. The number of objectives on which we would like to ascertain level of attainment of students would be very large. NCERT, 2005 specifies the following objectives with the complexity indicated in each area by the end of class V in whole numbers.

Number

- Knows number sequence up to 1000.
- Counts in different ways starting from any number.
- Reads and writes three-digit numbers.
- Expands a number with respect to place values.
- Compares numbers.
- Finds place value in numbers beyond 1000.
- Forms greatest and smallest numbers given 2 or 3 specific digits.

Addition & Subtraction

Adds and subtracts numbers by writing them vertically without regrouping and with regrouping.

- Uses the place value in standard algorithm of addition and subtraction.
- Solves addition and subtraction problems in different situations presented through pictures and stories.
- Frames problems for addition and subtraction facts.
- Estimates the sum and difference between two given numbers.
- Adds and subtracts single digit numbers and two digit numbers mentally.
- Adds and subtracts multiples of 10 and 100 mentally.
- Estimates sums and differences, and verifies using approximation.
- Appreciates the role of place value in addition and subtraction algorithms.

Multiplication

- Writes multiplication facts.
- Writes tables up to 10 × 10.

- Multiplies two and three digit numbers using lattice algorithm and the standard (column) algorithm.
- Frames word problems.
- Estimates product of given numbers.
- Completes multiplication facts by adding partial products, mentally e.g. 7 × 6 = 5 × 6 + 2 × 6
- Appreciates the role of place value in multiplication algorithm.

Division

- Divides a given number by another number in various ways such as by drawing dots, by grouping by using multiplication facts and by repeated subtraction.
- Explains the meaning of factors and multiples.
- Uses informal and standard division algorithms.
- Estimates quotients and verifies using approximation.
- Applies the four operations to life situations.

II-DIAGNOSIS OF WEAKNESSES

we would restrict to remedial teaching for weaknesses in minimum essentials such as labeling and writing of mathematical symbols - numerals 1-1000, >, <, =, +, -, ×, ÷, basic facts -addition and multiplication of one digit numbers and their subtraction and division corollaries, execution of procedures of addition, subtraction, multiplication and division of complexity specified by NCERT syllabus and adequate understanding of concepts and procedures.

Informal Procedures for Diagnosis of Weaknesses

The weaknesses may be identified informally on the basis of experience of earlier classes, questioning, observations, students' self-reports and report of teachers in earlier classes.

Formal Procedures for Diagnosis of Weaknesses

The weaknesses may be identified formally by the giving tests.

Diagnostic Tests

Diagnostic test in whole numbers have usually been restricted to difficulties in execution of procedures based on the established types of difficulties. But for planning remedial teaching for group teaching there is need to procure evidence on

- Prerequisites unless available on a recent test
- Accuracy and automaticity of basic facts
- Labeling and writing of mathematical symbols and terms
- Execution of procedures beginning from low to the desired complexity
- Understanding of concepts
- Understanding of procedures

Prerequisites

A prerequisite is a task which is necessary or simplifies the learning of a target objective. For example, knowledge of basic facts is essential for learning addition of multi-digit numbers. Understanding of associative law is a supportive prerequisite as it facilitates verification and simplification of some computations or doing them mentally. There is a need to get evidence on essential prerequisites unless available on a mastery test given recently. As failure to answer correctly many items relating to different objectives, may be due to lack of mastery of prerequisites. For example the prerequisites for different procedures are given below:

- Addition- Reading and writing of numbers, concept of place value and addition, addition facts, complex addition facts (addition of one-digit numbers to two-digit numbers)
- Subtraction – reading and writing of numbers, concept of subtraction and place value and subtraction facts
- Multiplication – reading and writing of numbers, concept of multiplication, distributive law and place value, multiplication facts and addition of multi-digit numbers
- Division – Reading and writing of numbers, concept of division and place value, subtraction of numbers, multiplication facts, concept of multiples and rounding of numbers to nearest ten

Mastery tests for knowledge and skills

There is need for a mastery test after completion of each unit dealing with minimum essentials viz. knowledge of mathematical symbols and vocabulary, automaticity of number facts and execution of procedures for multi-digit numbers of desired complexity.

Knowledge of mathematical symbols and vocabulary

It is necessary to check students familiarity with symbols and terms to ensure understanding of the questions, the students may be asked to read and write symbols and numbers given orally in random order, 0-1000, reading of statements involving +, -, ×, ÷, >, <, =, terms used for different numbers in an operation e.g. addends and sum in addition; minuend, subtrahend and difference in subtraction; multiplicand and multiplier or factors and product; in multiplication and dividend, divisor, quotient and remainder in division. Multiple mastery tests for reading and writing of numbers with2, 3, 4,…digit can be generated by the following websites which are free

- http://www.mathworksheetwizard.com/arithmetic.html
- http://www.math-drills.com/
- http://www.softschools.com/math/worksheets/
- http://www.mathworksheetwizard.com/

Automaticity of number facts

- Basic number facts are essential prerequisites for procedures for multi-digits numbers which in turn are essential component skills for many other objectives such as applications and problem solving. It is important that students acquire automaticity (answering without thought) in number facts and speed in procedures on multi-digit numbers of desired complexity so that they can concentrate on other aspects of applications and problem solving.

- Evidence on this may be sought by oral examination asking all the basic facts relating to an operation in random order and checking if the students answer them correctly and instantly or giving a sheet with all number facts relating to an operation in random order and asking them to write the answer if it comes immediately to their mind and mark X if they have to stop to think for an answer or count it in any way. It may be done for all operations and students asked to memorize the ones which they could not answer and tested on them.
- Multiple tests for addition, subtraction, multiplication and division facts can also be generated by printing worksheets from any of the websites given above.

Execution of procedures for multi-digit numbers

Multiple tests for execution of procedures of desired complexity can also be generated by printing worksheets from any of the websites given above.

Tests for understanding

There are degrees of understanding of concepts and different levels may be adequate at different stages of learning. The teacher would have to decide the desired level of understanding depending on the stage of learning of the student and complexity of a concept and procedure.

Understanding of concepts is a prerequisite for meaningful learning of procedures which are based on many facts, concepts and properties of concepts. For example, algorithm for multiplication of multi-digits numbers requires reading and writing of numbers, basic multiplication facts and their application to multiplication of even tens, hundreds etc., complex number facts (addition of a two digit number to a single digit number), addition of multi-digit numbers and concept of place value and distributive law. It assumes greater importance in group remedial teaching as it is not possible to look into individual misconceptions. Furthermore, when a student does not know the correct procedure, he still executes it using a buggy procedure e.g. subtract a larger number from a smaller number irrespective of whether the larger number is in minuend or subtrahend. Since students have been practicing buggy procedures for years, there is a need to teach these with understanding and provide sufficient practice so that students do not revert back to buggy procedures. This in turn requires evidence on present levels of understanding of concepts and principles to build on them.

A student understands a concept if he is able to

- represent a concept in many ways e.g. addition by manipulatives, pictures, number line, words, symbols, situations in daily life
- translate a concept from one mode of representation to another e.g. translate word problems to mathematical sentences, illustrate

mathematical sentences involving fundamental operations by manipulatives/ pictures, make up word problems for given computations.

- recognize its examples and non-examples e.g. which of the following are odd numbers 1, 4, 13, 16, 17
- generate an example with specified properties e.g. write largest/smallest number having digits 2, 6, and 9 using each digit once only
- recognize its non examples and explain why e.g. is 9 a prime number? If no, explain why?
- abstract its properties from many examples or illustrate properties given in words by giving examples e.g. give an example that shows numbers can be added in any order or derive commutative properly from many examples
- apply its properties for derivation of new number facts or simplification of computation e.g.
  $7 \times 9 = 9 \times 7 = 63$ (after they have learnt the table of 7)
  $8 + 7 = 8 + 2 + 5 = 10 + 5 = 15$
  $(179 \times 5) \times 2 = 179 \times (5 \times 2) = 1790$
  $483 \times 0 = 0$
- compare and contrast concepts by recognizing similarities and differences and inclusive relations between them e.g. difference in closure properties of even and odd numbers under addition, all odd numbers are whole numbers
- relate or integrate concepts with previously learned concepts e.g. subtraction is inverse of addition, division is inverse of multiplication and apply these for deriving subtraction facts and division facts respectively and verification of results

Understanding of procedures facilitates retention and its application to new situations and further learning. A students understands a procedure if he can

- use informal procedures and short algorithms.
- can explain steps in a procedure.
- can give reasons for steps in the procedure.
- can judge reasonableness of results by estimating results e.g. 298×7 should be close to 2100 or $40\overline{)756}$ should be between 10 and 20
- verify computational result by using relations between operations and using properties of operations e.g.
  minuend = subtrahend + difference
  subtrahend = minuend - difference
  dividend = quotient × divisor + remainder
  using associative property of addition of three or more numbers by adding, upside down and then in reverse order

using associative property of multiplication by expressing multiplier as product of two factors and then multiplying product of a multiplicand and a factor by the other factor e.g. 723 × 24 =(723 × 4) × 6.

- simplify computations by using distributive property of multiplication over addition or subtraction e.g. 548 × 39 = 548 × 40 – 548 × 1.
- apply procedures to situations in daily life.
- give situation in daily life in which a particular operation would be required.
- make stories for a given set of computations which may be judged in terms of applicability of the operation to the situation and reasonableness of numbers used.
- identify the correct fundamental operation for solving a word problem and solve problems requiring use of one, two or more fundamental operations
- recognize an operation given many examples of two sets of numbers which give a number and apply it to a new set of numbers e.g. If
  2, 3 → 6
  4, 7→28
  5, 8→40
  what fundamental operation is being used and apply the same operation to find what 7, 6 would give?

Evidence on these would facilitate planning of remedial teaching to enhance understanding of these.

Sequencing of Objectives

A recommended sequence of objectives for teaching and evaluation of numeration, notation, addition, subtraction, multiplication and division are given in Table 1 to 5. We have given it beginning with the most basic concepts for sake of completeness and use by primary school teachers who may select within each general objective of the table level of competency appropriate for their grade.

Selection of Objectives for Gathering Evidence

Evidence on objectives within a general objective may be sought by formal or informal evaluation beginning with a level where students in class VI seem to have difficulty. This may be based on evaluation of this or earlier classes or research studies in the area. The objectives which no one is likely to have attained due to lack of relevant experience may be excluded. Diagnostic tests have generally been restricted to weaknesses in execution of procedures based on established types of difficulties (Brueckner 1955, Buswell and Leonore, 1926, Gupta, 1971; Rastogi, 1991). These and studies by Mahajan (1995) and Institute of Advanced Studies in Education in Delhi

(1997) which included some evidence on understanding of concepts and procedures point to the need of getting evidence on the following in numeration system and different operations. The difficulty values of these are given in Appendix 3.

Numeration, notation and writing of numbers

- Reading and writing of numbers and translation of numbers given in figures to words and vice versa especially numbers with a nine in units place and numbers with zeros (in Hindi).
- Writing predecessor of numbers especially numbers with zeros in units place, both units and tens place; units, tens and hundreds place etc.
- Writing successor of numbers especially numbers with nine in units place; nine in units and tens place, units, tens and hundreds place etc
- Place values of specified digits in a number including some with zeroes.
- Places of specified digits in a number which may include zeroes.
- Comparison of numbers with different digits and same digits in different places.
- Writing smallest/largest numbers using specific digits.

Addition

- Automaticity of addition facts
- Addition of numbers including numbers with zeros in addends and different number of digits in addends.
- Explanation of steps in an example of addition.

Subtraction

- Automaticity of subtraction facts
- Subtraction of numbers especially those requiring renaming in two or more successive columns and with one, two or more successive zeros in the minuend.
- Explanation of steps in an example of subtraction.

Multiplication

- Automaticity of multiplication facts.
- Multiplication of multi-digit numbers with zero in multiplier or multiplicand.
- Explanation of steps in an example of multiplication.
- Conversion of units of measurement.

Division

- Automaticity of multiplication facts
- Division of numbers, solving problems involving both multiplication and division and finding percent of numbers.
- Explanation of steps in an example of division
- Application to problems in daily life requiring division.

TABLE 1

Sequence of Objectives for Evaluation of Numeration

Objective	Guidelines for assessment
Understands Concept of a number	
Numbers 0 to 10	How many objects are there in a collection of 1 to 5 objects or pictures of objects in any pattern and 6 to 10 objects or pictures of objects in a regular pattern. Check whether the students answer these instantly.
	Read numerals 1 to 10 in random order
	Write numerals 1 to 10 in a random order given orally.
	Count up to 10.
	Show n^{th} object (n =1 to 10) in a row of ten objects.
	Find the number of objects in a collection of objects or picture of objects by counting.
	How many specific objects are there in a place with no such objects and check whether the student says zero.
Numbers 10 to 1000	Read numbers 10-100 on a number line
	Count number of objects in a collection of 10-1000 objects given concrete representation of ones, tens, and hundreds.
	Read numbers 10 to 1000 given in random order.
	Write numbers 10 to 1000 given in random order given orally.
	Write numbers 0-1000 given in random order in words.
	Translate numbers 0-1000 given in words in random order in figures.
Understands Concept of Place Value	
Two-digit numbers	Count number of objects given 1-9 groups of ten objects e.g. bundles of sticks (ten-rupee notes) and 1-9 single sticks (one-rupee notes).
	Express numbers 1-99 in expanded notation.

	Given a number in expanded notation write the number. Round numbers to nearest ten
Three-digit numbers	Repeat the above for three-digit numbers with inclusion of objects representing hundreds and numbers with zero or nine at one and more consecutive places. Name the digits at different places Find place values of different digits including zero at different places Find successor (number that comes just after) of different numbers including numbers with nine at different places Find predecessor (number that comes just before) of different numbers including numbers with zero at different places Round numbers to nearest ten and hundred
Four digits numbers	Repeat these for five or six digit numbers except for number representation by concrete materials.
Compares Numbers	Compare two sets of objects having 1-9 objects. Recognize relations between numbers on a number line for a specific number and generalize it e.g. Is a specific number to the left (right) of A (a number marked on a number line) smaller (larger) than A. Would it be true of any number on the number line? What is the smallest number on a number line? Compare numbers using the concept of place value and sign of inequality, using numbers with different number of digits, same number of digits and same digits in different order. Make largest/smallest number using 2-4 specific digits
Develops Number sense	Round numbers to the nearest 10, 100… Estimate or write a number on a line segment whose end points are given. Estimate the number of beans in the jar and explain your procedure. Estimate the length a chain made by one thousand clips and explain your procedure.

TABLE 2

Sequence of Objectives for Evaluation of Addition

Objective	Guidelines for Assessment
Has concept of addition	Pick up two collections of a specific number of objects (1-9) and keep them aside, combine the two and tell how many objects are there.
	Read addition statements e.g. 4 + 8 = 12. Can you read it in another way?
	Pick up a collection of a specific number of objects (1-9) and add (1-9) more objects to it. How many objects do you have now? Express the above in symbols.
	Find addition facts using counting on.
	Add three or more one digit numbers by counting on.
	Solve word problems requiring addition of two one digit numbers.
	Make word problems for a specific addition statement.
	Find addition facts using number line.
	Read addition statements shown on a number line.
	Add mentally and explain how you got the answer: 50 + 30, 23 + 35, 56 + 75
Understands properties of addition	
Commutative	Fill in blanks 8 + 7 = 7 + ___ 5 + ___= ___ + 5
	Add 3 + 8 aloud and note if the student uses counting on from 8.
	Add 5 + 26 aloud and note if the student uses counting on from 26.
Associative	Explain what does (5 + 4) + 2 mean.
	Fill in blanks (5 + 4) + 7 = 5 + (4 + ___) (3 + 6) + ___ = 3 + (6 + ___)
	Find a short cut to add and add mentally (26 + 7) + 3

Add 347 + 73+184. How can you verify its answer, so that you do not make the same mistakes in number facts again?

Closure If you add two natural/whole numbers, do you always get a natural/whole number? If no, give an example in which it does not hold.

Adds 0 Identity Show 4 sticks in your left hand and no sticks in your right hand and ask how many sticks I have in my left hand, right hand and both hands? Express it in symbols.

Write a number for □ so that

4 + 0 = □

6 + □ = 6

179 + □= 179

Knows number Facts Ask addition facts in random order and note the addition facts in which the student counts on fingers or answers wrongly or give the student a sheet in which all addition facts are written in a random order and ask the students to answer the ones he/she can without thinking and place an X on others.

Adds Numbers Add using concrete materials/ expanded notation/short algorithm and explain steps in the procedure and reasons for them

$$\begin{array}{r} 50 \\ +\ 30 \\ \hline \end{array} \qquad \begin{array}{r} 70 \\ +\ 20 \\ \hline \end{array} \qquad \begin{array}{r} 40 \\ +\ 40 \\ \hline \end{array}$$

Two digit numbers that do not require renaming

$$\begin{array}{r} 45 \\ +\ 23 \\ \hline \end{array} \qquad \begin{array}{r} 56 \\ +\ 32 \\ \hline \end{array} \qquad \begin{array}{r} 35 \\ +\ 4 \\ \hline \end{array}$$

Three digit numbers that do not require renaming

$$\begin{array}{r} 325 \\ +\ 164 \\ \hline \end{array} \qquad \begin{array}{r} 346 \\ +\ 31 \\ \hline \end{array} \qquad \begin{array}{r} 473 \\ +\ 4 \\ \hline \end{array}$$

Three one or two digit numbers that do not require renaming

$$\begin{array}{r} 12 \\ 41 \\ +\ 33 \\ \hline \end{array} \qquad \begin{array}{r} 14 \\ 71 \\ +\ 10 \\ \hline \end{array} \qquad \begin{array}{r} 2 \\ 34 \\ +\ 3 \\ \hline \end{array}$$

Two or three digit numbers that require renaming from ones to tens

$$\begin{array}{r} 37 \\ +\ 46 \\ \hline \end{array} \qquad \begin{array}{r} 243 \\ +\ 308 \\ \hline \end{array} \qquad \begin{array}{r} 278 \\ +\ 13 \\ \hline \end{array}$$

	Two or three digit numbers that require renaming from ones to tens and also from tens to hundreds $\begin{array}{r} 376 \\ \underline{+\ 185} \end{array}$ $\begin{array}{r} 248 \\ \underline{+\ 75} \end{array}$ $\begin{array}{r} 743 \\ \underline{+\ 188} \end{array}$
	Round these to the nearest hundred and estimate the answer and check if the answer is reasonable.
Labels the terms	In 48 + 26 = 74, state the addend and sum
Applies the Procedure	Add money, length and capacity requiring no renaming of units Add money, length and capacity requiring renaming of units e.g. paise to rupee, centimetres to metres and millilitres to litres Solve word problems in everyday life requiring addition and having extraneous or missing data
Develops number Sense	Make up word problems that require specific computations. Evaluate these on appropriateness of the situation and reasonableness of numbers used. Judge reasonableness of answers by rounding off e.g. 893 + 207 should be about 1100.

TABLE 3

Sequence of Objectives for Evaluation of Subtraction

Objectives	**Guidelines for Assessment**
Understands concept of Subtraction	Set aside a specific number of object and take away certain number of objects How many are left? Express the above in symbols. Ask students to set aside two collections of specific number of objects (1 to 18) and (1 to 9) so that difference is a one-digit number. Ask them which one has more and how many more. Set aside 1 to 9 objects. How many should be added to it so that it would have a specific number of objects (2 to 18). Read 9 – 4, can you read it in another way? Use a number line to find a basic subtraction fact. Read a subtraction statement shown on a number line. Solve word problems requiring basic facts and different concepts of subtraction viz. separating from, adding on or comparing. Make up word problems on basic subtraction facts. Subtract mentally and explain how you got the answer: 80 - 40, 57 - 34, 67- 29.
Properties of subtraction	
Closure	If we subtract two whole numbers do we always get a whole number? If not, give a counterexample.
Commutative property	Is 5 - 3 = 3 - 5?
Associative property	Is (7 - 4) - 2 = 7 - (4 - 2)?
Subtractionof Zero	If I have 6 sticks and take away no sticks how many sticks would remain? Express it in symbols. Write a number for □, so that 6 - 0 = □

	4 – □ = 4 3 - 3 = □
Subtraction is inverse addition	State subtraction facts related to an addition of If 7 + 4 = 11, what is 11-7 and check if they answer instantly or not. Verify the computation given below by using another procedure which would involve different operation or use of subtraction facts. 740 – 226 = 514
Subtraction facts	Ask all basic subtraction facts and note down the ones he does not answer instantly or accurately or give the students a sheet containing all subtraction facts in random order and ask him write the answer it comes immediately to his mind and put an X if he has to count in some way.
Execute subtraction	Subtract using concrete materials/expanded notation/short algorithm and explain steps in the procedure and reasons for them in the sequence even tens and even hundreds: $\begin{array}{r}80\\ \underline{-40}\end{array}$ $\begin{array}{r}700\\ \underline{-300}\end{array}$ numbers requiring no renaming $\begin{array}{r}57\\ \underline{-23}\end{array}$ $\begin{array}{r}63\\ \underline{-30}\end{array}$ $\begin{array}{r}47\\ \underline{-\ 4}\end{array}$ one or two digit number from a two digit number requiring renaming $\begin{array}{r}54\\ \underline{-18}\end{array}$ $\begin{array}{r}48\\ \underline{-\ 9}\end{array}$ $\begin{array}{r}40\\ \underline{-28}\end{array}$ one, two or three digit number from a three digit number requiring renaming from tens to ones or hundreds to tens $\begin{array}{r}342\\ -257\end{array}$ $\begin{array}{r}378\\ -\underline{87}\end{array}$ $\begin{array}{r}240\\ \underline{-\ \ 6}\end{array}$ a two or three digit from a three digit number with zero in tens column $\begin{array}{r}504\\ \underline{-\ 87}\end{array}$ $\begin{array}{r}403\\ \underline{-\ 21}\end{array}$ $\begin{array}{r}701\\ \underline{-44}\end{array}$

	Round these to the nearest hundred and estimate the answer. Are the answers to the above questions reasonable? a two or three digit from a three digit number with zeros in both tens column and units column $\begin{array}{r} 800 \\ \underline{-\ 400} \end{array}$ $\begin{array}{r} 700 \\ \underline{-\ 247} \end{array}$ $\begin{array}{r} 600 \\ \underline{-\ \ 54} \end{array}$ Round these to the nearest ten and estimate the answer. Are the answers to the above questions reasonable?
Labels the terms	In 54 – 26 = 28, state the difference, subtrahend and minuend
Applies the procedure	Subtract money, length and capacity requiring no renaming of units Renaming of units e.g. rupee to paise, metres to centimeters and litres to millilitres Solve word problems in everyday life requiring subtraction both addition and subtraction subtraction and having extraneous or missing data
Develops number Sense	Make up word problems that require specific computations. Evaluate these on appropriateness of the situation and reasonableness of numbers used. Judge reasonableness of answers by rounding off e.g. 893-207 should be about 700.

TABLE 4

Sequence of Objectives for Evaluation of Multiplication

Objectives	Guidelines for Assessment
Understands concept of multiplication	Find the number of objects if certain groups (1 to 9) having equal number of objects (1 to 9) are combined. Express it in symbols. Find basic multiplication facts by repeated addition. Read a multiplication statement in horizontal or vertical form. Use a number line to find basic multiplication facts. Read basic multiplication facts on a number line. Represent basic multiplication of two one-digit numbers by area model. Read multiplication of two one-digit numbers represented by area model. Represent or read multiplication of two two-digit numbers by area model. Read multiplication of two one-digit numbers from a multiplication table.
Properties of multiplication	
Commutative	Fill in the blanks 4 × 7 = 7 × ___ 6 × 7 = __ × 6 After the student has multiplied two specific numbers, ask him to multiply the same numbers in the reverse order and check if the student answers instantly.
Associative	Explain what does (7 × 4) × 3 mean? Fill in the blanks (5 × 4) × 3 = 5 × (__× 3) (7 × __) × 6 = 7 × (__ × 6)

	Find a short cut and multiply mentally (26 × 5) × 2 Verify 74 × 12 using a procedure that makes use of different multiplication facts to avoid making the same mistakes again.
Closure	If you multiply two natural whole numbers, do you always get a natural or whole number?
Multiplication by 1	If you combine 7 groups of one object, how many objects you would have? Fill in the blanks 6 × 1= __ 8 × __= 8 748 × 1 = __
Distributive property	Fill in the blanks 5 × 6 = (5 × 2) + (5 ×__) __× 7 =__× 2 + __ × 5 Illustrate the property using an area model e.g. Show by using an area model (7 × 4) + (7 × 2) = 7 × 6 Use it to simplify computations e.g. find 43 × 758 - 43 × 58.
Knows Multiplication facts	Ask all basic multiplication facts in a random and note whether student answers them correctly and instantly or give the students a sheet in which all multiplication facts are written in random order and ask the student to write the answer if it immediately comes to his/her mind and put an X if he has to count or say the table of the number beginning from one.
Executes multiplication	Ask him to multiply and explain steps in the procedure and reasons for those using concrete materials expanded notation concept of place value and distributive property for the following: even tens, hundreds or thousands by one digit numbers 7 × 10, 4 × 100, 3 × 1000 a two digit number by a one digit number, requiring no renaming $\begin{array}{r}32\\ \underline{\times\ 3}\end{array}$ $\begin{array}{r}43\\ \underline{\times\ 2}\end{array}$ $\begin{array}{r}22\\ \underline{\times\ 4}\end{array}$

a two digit number and a one digit number requiring renaming

$$\begin{array}{r} 23 \\ \underline{\times\ 4} \end{array} \qquad \begin{array}{r} 46 \\ \underline{\times\ 7} \end{array} \qquad \begin{array}{r} 23 \\ \underline{\times\ 9} \end{array}$$

a three digit number with zero in tens column by a one-digit number e.g.

$$\begin{array}{r} 603 \\ \underline{\times\ \ 5} \end{array} \qquad \begin{array}{r} 307 \\ \underline{\times\ \ 9} \end{array} \qquad \begin{array}{r} 102 \\ \underline{\times\ \ 4} \end{array}$$

even tens by even tens e.g. 20 × 30, 40 × 60, 80 × 70.

a two digit number by a two digit number requiring renaming e.g.

$$\begin{array}{r} 58 \\ \underline{\times\ 23} \end{array} \qquad \begin{array}{r} 76 \\ \underline{\times\ 49} \end{array} \qquad \begin{array}{r} 43 \\ \underline{\times\ 56} \end{array}$$

Estimate these to check if the answers are reasonable.

Even tens or hundreds by even tens or hundreds e.g.

20 × 400, 340 × 50, 700 × 500

A three digit number by a two digit number e.g.

$$\begin{array}{r} 324 \\ \underline{\times\ \ 35} \end{array} \qquad \begin{array}{r} 478 \\ \times\ \underline{96} \end{array} \qquad \begin{array}{r} 756 \\ \times\ \underline{74} \end{array}$$

A three digit number by a three digit number with out zero in the multiplier

$$\begin{array}{r} 284 \\ \underline{\times\ 386} \end{array} \qquad \begin{array}{r} 725 \\ \times\ \underline{196} \end{array} \qquad \begin{array}{r} 613 \\ \times\ \underline{754} \end{array}$$

A three digit number by a three digit number with zero

in tens column

$$\begin{array}{r} 482 \\ \underline{\times\ 506} \end{array} \qquad \begin{array}{r} 782 \\ \times\ \underline{207} \end{array} \qquad \begin{array}{r} 329 \\ \times\ \underline{408} \end{array}$$

Estimate the answers to these by rounding off to the nearest hundred and check if the answers are reasonable.

Labels terms	In 32 × 5 = 160, which number is the, multiplier, multiplicand and product?
Applies the procedure	Ask the students to solve word problems in every day requiring multiplication of numbers or measures. multiplication and addition multiplication and subtraction multiplication, addition and subtraction multiplication with missing or extraneous data Make up word problems requiring multiplication. How many millilitres are there in 4 litres and 500 millilitres? How many centimetres are there in one metre and 20 centimetres? How many paise are there in 5 rupees and 70 paise Multiply a set of measures by a number e.g. rupees paise, metres and centimetres. Rupees paise m cm 1 75 3 15 × 5 × 6
Develops number Sense	Ask students to make up word problems that require multiplication of a specific number or measure by another number and judge these on appropriateness of the situation and reasonableness of numbers used. Judge reasonableness of answers by rounding off e.g. 478 × 20 would be about 800/1000/ 8000/10,000.

TABLE 5

Sequence of Objectives for Evaluation of Division

Objectives	Guidelines for Assessment
Understands Concept of Division	Set aside a specific number of objects (product of two one-digit numbers) say 12 and distribute these equally into 4, groups How many objects would each group have? Express it in symbols. If you want to give two objects to each student, how many students can you give it to? Express it in symbols. Read a division fact e.g. 15 ÷ 5 = 3 or $\begin{array}{r}4\\4\overline{)16}\end{array}$ Find basic division facts using repeated subtraction. Find basic division facts using a number line. Read basic division facts shown on a number line. Read basic division facts from a multiplication table Solve word problems in daily life requiring the measurement concept of division (find number of groups given size of the group). the partitive concept (given the number of groups find the size of each group) Make stories for given basic division facts. Divide mentally and explain how you got the answer: 50 ÷ 5, 63 ÷ 6, 42 ÷ 5 Write all factors of 12. Fill in the blanks: If 7 × 8 = 56, 7 and 8 are_____ of 56.
Properties of division	
Closure	Can we always divide two whole numbers? If no, give an example in which it is not possible.
Commutative property	Is 5 ÷ 3 = 3 ÷ 5?
Associative property	Is (24 ÷ 6) ÷ 3 = 24 ÷ (6 ÷ 3)?

Division by 1

Fill in blanks
7 ÷ 1 = __
8 ÷__ = 8
4 ÷ __ = 1

Distributive property

Fill in blanks
25 ÷ 5 = (20 ÷ 5) + (__÷ 5)
___÷ 4 = (40 ÷ 4) – (4 ÷ 4)

Division is inverse of multiplication

Ask them to state division facts related to a multiplication fact e.g., after students answer 8 × 7 = 56, ask what is 56 ÷ 8 or 56 ÷ 7 and check if they answer instantly or not.

Knows division facts

Ask all basic division facts in random order and note down the ones the student does not answer instantly or accurately or give the students a sheet containing all division facts in random order and ask him to write the answer if he can think of it instantly and mark an X if he can not.

Execute division

Ask him to divide by using concrete material long and short algorithm and explain steps in the procedure and give reasons for them.

Divide even tens, hundreds thousands by even tens and hundreds without a remainder, e.g.

$20\overline{)60}$ $400\overline{)800}$ $50\overline{)250}$

even tens and hundreds by a single digit without a remainder e.g.

$5\overline{)50}$ $3\overline{)900}$ $4\overline{)800}$

one digit divisor and one digit quotient with a remainder

$2\overline{)9}$ $5\overline{)48}$ $7\overline{)23}$

one digit divisor, two or three digit dividend, two digit quotient and a remainder

$7\overline{)85}$ $2\overline{)53}$ $6\overline{)95}$

one digit divisor, three or four digit dividend, three digit quotient with a zero at hundreds, tens or units place

$8\overline{)850}$ $3\overline{)6034}$ $4\overline{)121}$

Two digit divisor, one or two digit quotients estimation yields quotient which is correct e.g.

$28\overline{)94}$ $56\overline{)857}$ $86\overline{)682}$

	Two digit divisor, one or two digit quotient, estimation yields quotient which is too large or small. $46\overline{)947}$ $34\overline{)671}$
	Two digit divisor, one or two digit quotient with 0 in the quotient $63\overline{)681}$ $34\overline{)701}$ $22\overline{)671}$
	Simplify questions involving both multiplication and division. $\frac{12\times100}{4}$ $\frac{9\times100}{30}$ $\frac{50\times40}{20}$
Label terms	In the division given below name the dividend, divisor, quotient and remainder
Verify results	Verify answer to a division problem using the relationship; Divisor = divisor × quotient + remainder
Applies the procedure	Solve word problems in every day life requiring division division and addition division and subtraction division and multiplication division and any two other operations. division with extraneous or missing data divide measures of money, length, and capacity, time by a number requiring no renaming of units divide measures of money, length and capacity by a number requiring renaming of units rupees to paise, metres to centimetres and litres to millilitres.
Develops number Sense	Make up word problems requiring specific computations Evaluate these on appropriateness of the situations and reasonableness of numbers used. Judge reasonableness of answers by rounding off e.g., 8100 ÷ 90 is about A 90, B 900, C 9000.

III-REMEDIAL TEACHING

After weaknesses of individual or group have been identified we need to make decision about remedial teaching, think about its causes and plan remedial teaching based on our conjectures about causes.

Causes of Weaknesses

It is helpful to keep in mind basic causes of student's weaknesses which would facilitate making conjectures in particular cases.

The causes of an individuals difficulty in learning may be physiological such as visual or auditory handicap, emotional such as deep seated anxiety or fear or hatred of mathematics, or problems at home like discord between parents, alcoholic father, poor home environment - lack of encouragement and facilities for studying at home, need to earn or help parents in their vocation after school, low intelligence or pedagogical reasons. These need to be examined for making a conjecture and planning remediation by appropriate means such as medical check-up and aids for physical handicaps, counseling for emotional stress, providing facilities to compensate for poor home environment, realistic expectations from children of low intelligence and remedial teaching for pedagogical causes. However, such remediation is not feasible at present due to paucity of resources. We will confine ourselves with remediation of weaknesses of group due to pedagogical causes with some provision for individual differences to ensure mastery of minimum essentials.

Pedagogical causes may relate to the syllabi for different grades, teaching - learning activities, and evaluation. As discussed in chapter 1, for most teachers textbook is the curriculum and at best they teach what is given in the textbook. As all government schools in Delhi and a large number of schools in other parts of the country use NCERT textbooks. The deficiencies in the textbook discussed earlier are predominant causes. These deficiencies are likely to be reflected in teaching due to nearly total dependence of teachers on textbook. A fortiori as all subjects up to class V are taught by the same teacher. Most of these teachers do not have good understanding of mathematical concepts and procedures themselves nor have access to teaching-guides or books on teaching of mathematics. They are not likely to make up for deficiencies in the textbook.

Lack of assurance for mastery of minimum essentials is another very important reason. Mathematics learning largely consists of building understanding of concepts onto already understood concepts.

Making Decisions for Remedial Teaching

The overall performance of students on diagnostic tests may be used to identify students that need remedial teaching. Retraining needs for these students may be ascertained from the type of errors made or item missed which may be supplemented by oral interview to pinpoint the cause of error or difficulty.

The proportion of students answering different items correctly called its difficulty or facility or p value should also be determined for each item. This would help in identification of areas in which the whole class or group needs remedial teaching.

The decision about areas in which remedial teaching should be done calls for desired performance standards. Minimum levels of learning require that difficulty value of items seeking evidence on an objective should be 0.80 or higher for the item and a student should be able to answer correctly 80% or more of the items seeking evidence on the objective (NCERT, 1986, p.9). However, they have not provided any tools for it. We feel it is not possible to achieve these standards for all types of objectives in view of their nature (minimum essentials or developmental) and enabling conditions at present or the best we can hope to provide for in the near future. The standards should be based on complete mastery of minimum essentials allowing for errors of measurement and other reasons for weaknesses such as low aptitude, physical disability, emotional problems in case of some students. Standards for minimum essentials such as learning of writing of symbols such as numerals, +, -, ×, ÷, =, >, <, basic number facts, knowledge of steps in a procedure and rationale for these, execution of procedures of desired computational complexity may be set at 80-90% or more correct for 80-90% of the students. The standards for higher level objectives like understanding of concepts and procedures should be set as high as attainable under empirical reality of enabling conditions using teachers' judgments or standards attained by top 10 to 25 percent of the students. These would differ from school to school.

If the difficulty value of items is below these standards, teaching may be planned to raise these to desired standards.

Organization of Remedial Teaching

Remedial teaching can be organized at several levels by the teacher, school or directorate depending on whether it is after teaching a lesson or topic or before a stage of education. It may be conducted for individuals, small groups or large groups depending on the number of students who need it and resources.

As remedial teaching is a felt need in almost all government schools for most of the students and present resources rule it out for individual or small groups or independent learning by students under teacher supervision due to lack of learning materials and reading skills of students, we would discuss it in the context of large groups by the teacher.

It would be desirable that Directorate or schools organize remedial teaching for class VI during summer holidays. It has several advantages:

The teacher and students can give more time during holidays to teaching and learning respectively.

The students can be grouped into homogeneous groups on the basis of their overall score on a pretest on whole numbers or even topics within it.

Space necessary for group activities or use of manipulatives would be available.

Teaching outside school hours also has some of these advantages but it may not be practical as most government schools have double shifts or some children commute by school buses.

If teaching outside school hours is not possible, it may be conducted during mathematics classes at the beginning of the year. The individual differences would pose a more serious problem unless sections can be formed on the basis of the performance in mathematics. The problem of individual differences exists even for homogeneous groups as students with the same overall score may have weaknesses in different aspects.

Guidelines for Remedial Teaching

We feel the following guidelines would enhance effectiveness of remedial teaching.

Motivate students

Motives largely determine what the students learn. Students may be motivated to learn by:

- establishing a favourable emotional climate by being encouraging, avoiding sarcasm and patronisation
- making students feel part of the program by seeking active cooperation and giving them responsibility for learning and evaluation
- discussing the utility of mathematics in everyday life
- asking questions that stimulate thinking such as how do you, a housewife, a farmer, a shopkeeper uses estimation, different fundamental operations in daily life
- providing opportunities for success by ensuring mastery of prerequisites
- presenting subject matter through small increments and seeking active participation by all students
- Base it on experiential background of students. These students have much wider experiences, larger vocabulary and language skills and know

something about all concepts and procedures that are taught than students when these concepts are introduced first. They may practice addition and subtraction facts together for automaticity using relations between them e.g. 7 + 5 = 12 => 5 + 7 = 12, 12 - 5 = 7, 12 - 7 = 5. Similarly multiplication and division facts may be practiced together.

Fact families for addition and subtraction and multiplication and division are given in Appendices 2 and 3 respectively.

If use of manipulatives is not practical, it can be replaced by oral explanations using analogy with money.

But they also pose special problems due to history of learning in a rote manner and use of buggy procedures which differ from student to student.

There is thus a need to teach concepts and procedures with understanding for remediation of different misconceptions and provide enough practice to ensure that they do not revert back to buggy procedures.

Provide for mastery of minimum essentials

As mathematics learning largely consists of building understanding of concepts onto already understood concepts. All students should have mastery of minimum essential, like labeling and writing of numerals, +, -, ×, ÷, =, >, <, basic number facts and execution of procedures of desired difficulty. This may be ensured by:

- adopting a pace that does not frustrate
- ensuring mastery of prerequisites
- ensuring understanding of concepts and rationale of procedures before teaching the short algorithm
- using direct teaching of skills (Silbert et al, 1990)- it is akin to programmed learning except that instead of students reading the frames at their own pace, teacher asks the questions to which students are expected to respond in unison when the teacher gives the signal
- providing sufficient practice beginning with easy skills and emphasizing numbers which students find difficult
- monitoring progress continuously by providing answers, diagnosing difficulties of those who could not answer or got wrong answer, remediation of those and providing more practice by giving home work referenced to their difficulties. Items given in Tables 1 to 5 using other numbers or situations may be given.
- giving responsibility for ensuring automaticity of number facts and mastery of procedures to students.

Base it on diagnosis of weakness

Diagnostic tests based on hierarchy of concepts and procedures given in Tables 1 to 5 may be given in a content area before undertaking remedial teaching. Evidence on objectives which have been mastered by most of the students or no one on the basis of informal evaluation or experience of earlier classes may be excluded. It is better to err on seeking evidence on easier objectives as there may be surprises. For example, we were surprised to find that many class VI students could not write numbers up to 99 when given orally in Hindi, while they could add and subtract these correctly. These tests may be administered to all students. The tests on understanding of concepts and procedures should be given under power conditions while those on number facts and execution of procedures should be timed. These tests may then be scored.

The difficulty value of items - the proportion of students who got the item correct may be determined for each item. This will point to the strengths and weaknesses of the class as a whole. Remedial teaching should be based on these.

The students' scores may be used to divide them into three groups on the basis of overall performance. The students with very low scores may be given special attention by asking them to sit in front so that teacher can diagnose their difficulties and provide remediation for them and give individualized home work referenced to their weaknesses. The student with high scores may be given individualized home work to enhance understanding of concepts or problem solving or asked to help weak students.

Emphasize understanding of concepts and procedures.

Teaching for understanding of concepts in mathematics with special reference to upper primary stage was discussed in Mahajan (1997). We will confine our discussion here to concepts in whole numbers.

Understanding of concepts

It may be enhanced by

- presenting a concept in many ways such as real world situations, manipulatives, pictures, spoken and written symbols e.g. concept of an operation say subtraction may be represented as

  taking away a specified number of objects from a larger number of objects and finding how many are left

  giving two groups of objects or pictures of objects find out which groups has more and how many more

  using situations in daily life

  presenting it on a number line
- providing opportunities for translation from one mode of presentation to another such as

  illustrate $12 \div 4 = 3$ by using objects or pictures of objects

  make a story for $8 \div 4$

- relating it to earlier learning e.g. division is inverse of multiplication and hence division facts can be found out from multiplication facts.
- abstracting properties of concepts such as commutative and associative properties of addition and multiplication, identify elements of these and distributive property of multiplication and division from many examples and enhancing understanding of these by asking students to
  recognize or give examples of these
  fill in blanks in examples of these e.g.
  5 + 8 = 8 + ___ (one correct answer)
  7 + ___ = ___ + 7 (many correct answers)
- derive number facts yet to be learned from known facts e.g.
  3 × 8 = 8 × 3
  12 × 7 = (10 + 2)×7 = 70 + 14 = 84
- simplify computations e.g.
  think of 9 + 7 = 9 + 1 + 6 = 10 + 6 = 16 instead of counting on
- recognize their applicability or otherwise to new situations and give a counter example if not applicable e.g.
  Is 8 - 5 = 5 - 8?
  If we subtract two whole numbers, would we always get a whole number?
  If no, give a counter example.

Understanding of procedures of multi-digit numbers

It may be enhanced by

- explaining rationale of different steps by first using manipulatives like Dienes blocks, sticks, paper money or verbal explanations using familiar objects like money, then expanded notation, long algorithm and finally short algorithm
- using properties of the operation for simplifying computations e.g.
  756 × 169 – 756 × 69 = 756 × (169-69) =75600
- Verifying results by using relations between operations and properties of operations e.g.
  minuend = subtrahend + difference
  dividend = divisor × quotient + remainder
  43 × 39 = 43 × (40 -1) = 43 × 40 - 43
- estimating results to judge reasonableness of results e.g.
  789 × 3 should be about 2400
  400 - 295 should be close to 100
- solving problems in everyday life requiring an operation
- making up stories for given computations
- applying it to make connection within mathematics and other subjects

Prepare for further learning

This may be accomplished by emphasizing or introducing activities which would facilitate further learning such as

- studying properties of system of whole numbers and constraints which call for its extension to more inclusive number systems
- introduction of number line for representation of whole numbers and operations on them which can then be used for representation of integers, rational numbers, real numbers and operations on them to facilitate their understanding or introducing area models for multiplication which can then be used for enhancing understanding of distributive property of multiplication over addition, and multiplication of decimals, fractions and product of sum and differences of literal numbers.
- study of patterns and relations which would later be helpful in studying algebra.

Provide systematic review

The students tend to forget and revert to earlier misconceptions. There is a need to provide distributed practice after students have attained mastery of a procedure unless it is being reviewed as a component skill for a more complex skill or problem solving. The examples should be chosen to provide practice on numbers or number combinations on which most students had difficulty.

Evaluate continuously for guiding teaching and learning

The effectiveness of remedial teaching needs to be evaluated continuously for ensuring mastery of minimum essentials and adequate understanding of basic concepts and procedures. The type of errors needs to be noted rather than inaccuracy of answers. This may be done by evaluating homework, formal tests, observations, written records of class work and oral questioning.

Allow for adjustments and deviations suggested by evaluation and provide remedial teaching to those who have not yet attained mastery of minimum essentials by using an alternative approach or providing more practice.

Specific Suggestions for Remedial Teaching for Weaknesses

Reading and writing of numbers

The difficulty in reading and writing numbers up to 100 seems to be due to lack of aids for memorizing, sufficient practice and review. As each number in Hindi has a different name, the student has to learn one hundred names for numbers 1-100. It would be desirable to change the numbers in Hindi akin to English including teen number as suggested by Kapur (1969) viz "dus ek", "dus do",... "bees ek, "bees do" reducing these to twenty words. But until that is done, there is a need to teach learning strategies to learn these.

Numbers 1 to 100 may be arranged in a number chart given in Table 6. The sound pattern in different rows and columns can help in memorizing them. Most of the children in class VI can read numbers in a serial order and write those in first row and last column. Asking them to read all rows beginning with second up to last but two columns and tell it they notice a pattern in rhyming and any exceptions to that. They will notice all numbers in a row up to eighth column rhyme. It would then help them in identifying the digit on the left viz 1 if it rhymes with "rah", 2 with "ees", three with "tees", 4 with lees 5 with "van" or "pan" 6 with "sath", 7 with " utter" 8 with " asi" 9 with "nave". Provide practice on these. Then ask them to read or read with them all numbers in different columns beginning with second row and see if they notice something common in the first sound and exceptions if any.

Table 6

The Hundred Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

It can help them in identifying the digit in ones' place viz., 1 for "ik" except 11, 2 for "ba or bi", 3 for "te or ti", 4 for "chou", 5 for "pan" or "puch", six for "chh except 16, 7 for "sa" 8 for "ath ", 9 or "un"except 89 and 99. For the numbers with 9s in units place it may be explained that "un" means one less than the number in the last column with which it rhymes, eg., "unees" is one less than "bees", untees is one less than "tees', etc., whereas nvasi and "ninyanve" mean nine more that "asi" and "nabbe". Provide practice on these for retention. These features may be pointed while teaching serial counting also.

The number chart can also be used as an aid to learning number relations e.g., a number in any row is greater than all the numbers to its left in its row and rows before it. A number in any row is smaller than numbers to its right in that row and numbers in rows after it. Each number in a row is one more than the number just before it in the same row and the first number is one more than the number the last column in the row just before it A number which comes just after a number is called the successor of that number. Provide or ask for specific examples using the term successor with more emphasis on numbers with nine in units place. Similarly predecessor of a number may be explained.

Number line may be introduced which enhances understanding of whole numbers and facilitates learning of other number systems viz. system of integers, rational and real numbers. It can also be used to reiterate number relations viz. concept of zero as beginning of a number line and the smallest whole number. Each number is greater than all numbers to its left and one more than the number just preceding it viz. its predecessor and it is less than all numbers to its right and one less than the number which follows it next viz. its successor.

For use of concrete materials and teaching with understanding of other topics, see the textbooks for classes I to V and papers published by author on web

http://www.vidyaonline.net/download.php?ID225
http://www.vidyaonline.net/download.php?ID224
http://www.vidyaonline.net/download.php?ID223
http://www.vidyaonline.net/download.php?ID222
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http://vidyaonline.org/writeup/graph.pdf
http://vidyaonline.org/writeup/Cutting.pdf
http://vidyaonline.org/writeup/dotpaper.pdf
http://vidyaonline.org/writeup/sticks.pdf

In these books sticks have been used as concrete materials for teaching of whole numbers. If that is considered cumbersome, analogy with money may be used with these children assuming that only one-rupee, ten-rupee and hundred-rupee and thousand-rupee notes are available for the concept of place value and understanding of operations using direct teaching.

Direct Teaching using money

Concept of place value for writing larger numbers

It may be taught by asking questions to students building on what they know. The expected answers by student are given against them. It they cannot or give wrong answers, probe further or give the answer. Give ample time for thinking of the answers to weak students by asking all students to give the answer only after teacher gives a signal.

Teacher	Students
Teacher asks referring to a chart of Table 6 which may be hung in the classroom.	
Which digits have been used for writing all numbers 1 to 100?	0, 1, 2, 3,…,9
How may digits are used?	10
What is the successor of 9?	10
How does its writing differ from that of others in the row?	It has two digits

We write all two digit the numbers, using digits 0-9 only, a two digit number larger than 9 is written by using two places and digits 0 to 9. The place at the left is called tens place and the place at the right units place. One in 10 represents a ten and zero is written after it to distinguish it from one and indicates there are no units. One ten equals ten units; this is just like exchanging for one ten-rupee note for ten one-rupee notes. The successors of numbers are written by increasing the digit in units place by one up to nine. The successor of a number with nine in units place is written by increasing the digit in tens place by one. The increase by one in tens place is comprised of nine in units place and one more and thus no units are left which is denoted by zero. We continue in this manner for writing numbers up to 99. Successor of highest two digit number viz. 99, is written by using another place called hundreds place and one here denotes one hundred and zeroes in tens and units places indicate absence of tens and units.

Numbers beyond 100 may be written in serial order by successively increasing digit in units place by one up to nine viz. 101, 102, 103... 109 that are read as hundred one, hundred two, hundred three…and hundred nine respectively. Numbers after 109 are written by prefixing one before numbers 10 to 99 viz. 110, 111,112...199 and are read as one hundred ten, one hundred eleven, one hundred twelve…one hundred ninety nine. Increasing a number with nine in both units and tens place is accomplished by increasing the digit in hundreds place by one and writing zero in units and ten's place. Thus successor of 199 would be 200, successor of 299 would be 300. We can write numbers up to 999 in a similar manner. Teacher may provide practice on these by asking successor of numbers with emphasis on successor of number with even hundreds, numbers with 9s in units place and numbers with 9s in both units and tens place. Assure mastery of these by tests. Reading a three-digit number involves prefixing the name of the digit in hundreds place followed by hundred and reading of one or two digit number e.g. 796 as seven hundred ninety six and 207 as two hundred seven. Provide practice on reading and dictation of three digit numbers in random order with emphasis on numbers with zero in tens place and nine in units place and translation of these to words and those given in words in figures until they have mastery.

The successors and reading and writing of four and more digit numbers may be taught and tested in a similar manner.

Expanded notation of numbers and concept of place value

These may be taught by analogy with money. The students in class V, especially those from lower classes have facility with money and know equivalence of one ten-rupee note to ten one-rupee notes, equivalence of one hundred-rupee note to ten ten-rupee notes. As many of them do shopping and/or help parents in their trade. The students may be asked how many notes of one-rupee denomination and ten-rupee denomination you would need for a specific amount (Rs 10 to 99) if notes are available in denominations of one and ten only and you want to have the *smallest* number of notes. Most of them would be able to give number of ten and one rupee notes for different amounts e.g.

64 = 6 ten-rupee notes and 4 one-rupee notes
53 = 5 ten-rupee notes and 3 one-rupee notes
70 = 7 ten-rupee notes

We may also write these as

64 = 6 tens + 4 units
53 = 5 tens + 3 units
70 = 7 tens

How much money do you have if you have 6, 4, 7 ten-rupee notes? (60,40 and 70 respectively)

So we can also write these as

64 = 60 + 4
53 = 50 + 3
70 = 70

Thus 6 in tens place stands for 60 and is called the place value of 6. Any digit in tens place of a number has a place value equal to ten times its value. The place value of digit in units place is equal to it. Expressing a number as sum of its place values is called its expanded notation. Provide practice on writing expanded notation of numbers and writing numbers whose expanded notation is given including numbers with zero in units place. Provide opportunities for developing analytical reasoning by asking them to write smallest/largest two digit number, smaller/larger numbers with two specific digits.

Technique for comparison of two numbers may also be explained in terms of place value. Any number which has more tens is greater than a number with lesser number of tens. It the number of tens is the same, then any number with larger number of units is greater. Symbols > and < may be introduced and they may be asked to compare numbers using these with emphasis on comparison of numbers where students have difficulty e.g. 34 and 43 (same digits in different order) 9 and 90 (some think zero means nothing). Some students tend to forget whether > is a sign for greater than or less than. The corner or point where two lines meet always point to the smaller number with examples may be helpful in choosing the correct symbol.

Writing place of units above numbers may be introduced to facilitate remembering the places and their place values as that many units and tens.

Expanded notation of three digit numbers 100 to 999 may be taught in a similar manner by including hundred-rupee notes and its equivalence to ten-ten rupee notes. Following activities may be provided in order by asking questions on:

Number of notes of different denominations needed for specific amounts using smallest number of notes.

Money corresponding to specific number of hundred-rupee, ten-rupee and one- rupee notes each between 0 and 9.

Write and read numbers using a place value chart.

Expanded notation of numbers and numbers whose expanded notation is given including numbers with zero at tens place and both at tens and unit places

The place values of different digits in three digit numbers including numbers with zero at tens place and both units and tens place

Places of different digits in three digit numbers including numbers with zero.

Compare three digit numbers using concept of place value and symbols > and <.

Write numbers using concept of place value and analytical reasoning e.g. smallest, largest and all possible numbers given three specific digits using each digit once.

Write smallest and largest three digit numbers having three specific digits using each digit once only which may include zero also.

Expanded notation and place value of four digit numbers may be taught similarly by including notes of thousand - rupee denominations if necessary. Beyond this analogy with money may not be necessary.

Place values of 5-8 digits numbers may be taught by extending place value chart to the left and use of two places for thousands, lakhs, crores for their units and tens. The place of digits beginning with right are called units, tens, hundreds, thousands, ten thousands, lakhs, ten lakhs and crores with place values of 1, 10, 100, 1000, 10,000 1,00,000, 10,00,000, and 1,00,00,000 times the digit in these places respectively. Note each place has a value ten times the value of the place to its right and ten ten-thousands or hundred thousand equal a lakh, ten ten-lakhs or hundred lakhs equal a crore.

Reading and writing of large numbers is facilitated by inserting commas after a thousand for 4 or 5 digit numbers, after a lakh for 6 or 7 digit numbers and after a crore for 8-9 digit numbers. Practice on reading, writing and translation of numbers from figures to words and vice-versa may be provided and mastery assured as it is an important life skill and would be necessary even with access to calculators and computers. Numbers for examples and practice should include numbers with one and more zeros at different places.

Number sense may be developed by asking them to give examples of 4-6 digit numbers in daily life and estimate numbers in large collections e.g. numbers of students in morning assembly, length of a million clips laid side by side, number of beans in a jar, a number say 150 on a line segment whose end points are marked 0 to 500.

Conceptual Understanding of Fundamental Operations may be enhanced by asking students to

- solve word problems involving basic facts and various interpretations of an operation e.g. partitive and measurement division, separating, matching and adding on in subtraction
- make up stories for basic facts
- suggest situations in which an operation may be used
- write or tell how a farmer/house-wife/shopkeeper or students themselves use different mathematical operations in day-to-day life
- illustrate properties of operations given in words or abstract them from a number of examples
- recognize properties that do not hold and give a counter example
- use properties of operations for deriving number facts 6×8 from 8×6 etc. or simplifying computations e.g. to find $8 + 5 + 2$ first add 2 to 8 and then 5 to 10.
- use relations between operations to derive related facts e.g. $5 + 5 + 5 + 5 = 4 \times 5 = 20$, $13 - 7 = 6$ from $6 + 7 = 13$, $56 \div 8 = 7$ from $7 \times 8 = 56$
- apply number facts to add and subtract even tens, hundreds, thousands or multiply a one digit number by even tens, hundreds and thousands and derive corresponding divisions facts.
- finding out basic facts using number line
- reading number facts shown on a number line
- Area models may be introduced for multiplication as these also can be used to facilitate understanding of multiplication of two digits numbers, distributive law, multiplication of fractions, decimal, rational and literal numbers and combination of literal numbers with other numbers.

- Make an addition table e.g. Table 7
- Use addition table to

  find addition facts.

  find the rule of addition of zero which may be generalized to larger numbers.

  verify commutative law of addition for a few examples and generalize it.

  get many partitions of a number by drawing a diagonal with the same number in different columns.

  8 = 8 + 0, 7 + 1, 6 + 2, 5 + 3, 4 + 4, 3 + 5, 2 + 6, 1+ 7, 0 + 8

  10 = 10 + 0, 9 + 1, 8 + 2, 7 + 3, 6 + 4, 5 + 5, 4 + 6, 3 + 7, 2 + 8, 0 + 10

  find subtraction facts
- deduce relations between addition and subtraction facts by finding related facts for a few number combinations
- recognize patterns in the addition table e.g. between numbers in columns 1 and 4, 2 and 7 etc. and express it in words

TABLE 7

ADDITION TABLE

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

- Make a multiplication table-Table 8
- Use a multiplication table to
  find multiplication facts.
  generalize rules of multiplication by zero and one.
  deduce commutative law of multiplication.
  get many factorization of a number involving one digit factors e.g.
  24 = 6 × 4 = 4 × 6
  3 × 8 = 8 × 3
  note diagonal entries are special numbers viz. squares of numbers.
  find division facts by excluding the zero row and column as division by zero is not permissible.
  relate multiplication and division facts.
  verify a ÷ b = c, then a ÷ c = b using appropriate examples and generalize.
  verify distributive law from the table that is 7 × 6 = 7 × 4 + 7 × 2 and derive table of 11, 12 13…etc. by adding appropriate columns which sum to 11, 12, 13,…
  derive other facts from patterns in a column e.g. 9 × 7 is 7 more than 8 × 7 = 56.
  derive relations between numbers in two columns in which one is a multiple of the other e.g. columns 2 and 4, 2 and 8, 3 and 6, 3 and 9, 4 and 8, 5 and 10.

TABLE 8

Multiplication Table

×	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

Execution of operations on multi-digit numbers

Most students had difficulty with execution of division, many students with multiplication and subtraction especially those involving 0 in the multiplicand or multiplier and minuend respectively a few with addition. Most of the students can not explain reasons for steps in the procedure. The common response was to get the right answer. There is thus a need to do remedial teaching explaining reasons for steps in the procedure for all of these, assuring mastery of prerequisites and providing practice to students on execution of operations in which they are weak.

For use of sticks as concrete materials for understanding of the concept of place value, see books by the author given earlier, here we would discuss analogy with money with the restriction that it is available in denomination of one-rupee, ten-rupee, hundred-rupee and thousand-rupee notes only and the facility for exchange of notes of equal value for smaller or larger denominations. Reasons for steps in the procedure may be explained by using analogy with money and direct teaching (Silbert et al, 1990). Direct Teaching is akin to programmed learning except that instead of asking students to read the frames on his own and respond, these formats are presented by the teacher and all students are asked to respond in unison after teacher gives a signal. This is to ensure that all students have had time to think of the answer. The teacher corrects responses and gives explanations. Formats for explaining renaming in addition and subtraction of two digit numbers and multiplication and division by one digit numbers and their generalization to larger numbers may be presented as follows after ensuring mastery of prerequisites for these:

Addition - The following format may be used for developing for conceptual understanding of two-digit numbers with renaming which may then be generalized to larger numbers and addition of more than two numbers.

Teacher	Students
Suppose you have 2 ten-rupee notes and 5 one rupee notes. How much money do you have?	Rs.25/-.
Your mother gives you 3 ten-rupee notes and 8 one-rupee notes more to buy a book.	Rs.38/-
How much money do you have now? (If they say 63 ask in terms of ten–rupee and one-rupee notes)	5 ten-rupee and 13 one-rupee notes
How did you obtain it.	By adding tens to tens and ones to ones
Yes, we always add the tens to tens	

and ones to ones, in
general likes only.

What you may do to have the smallest number of notes?	Exchange ten one- rupee notes for one ten-rupee note
Then how many ten-rupees notes and one-rupee notes do you have?	6 ten-rupee-notes and 3 one- rupee notes
How much money is that?	Rs. 63

Similarly numbers may be added using expanded notation and renaming if the sum of units is greater than 9.

$$\begin{array}{l} \ \ \ 25 = 2\text{ tens} + 5\text{ units} \\ \underline{+\ 38 = 3\text{ tens} + 8\text{ units}} \\ \qquad = 5\text{ tens} + 13\text{ units} \\ \qquad = 5\text{ tens} + 1\text{ ten} + 3\text{ units} \\ \qquad = 6\text{ tens} + 3\text{ units} = 63 \end{array}$$

or you may write it in short form as

$$\begin{array}{r} 25 \\ +\ \underline{38} \\ 63 \end{array}$$

The number which are added e.g. 25 and 38 are called **addends** and 63 the number obtained by adding **sum**.

Similarly we may add numbers having three or more digits by arranging them vertically with units, tens and hundreds of all numbers aligned in the same column and add beginning with units. If the sum of units exceeds 9, we rename it as tens and units, write the units in unit's column for the sum and tens above the tens column, then we add the tens including these tens and if the sum exceeds 9 we change it into hundreds and tens. This is analogous to exchanging ten ten-rupee notes for a hundred-rupee note and write it above the place of hundreds to remember adding to hundreds and so on. For example,

$$\begin{array}{r} {}^{1}\ \ \ \ \ \\ 472 \\ +\ \underline{234} \\ 706 \end{array}$$

Similarly, three or more numbers with different number of digits may be added by arranging them in vertical form so that units, tens, hundreds, etc. of different numbers are in the same column and adding the numbers in a column beginning with units and renaming where necessary e.g.

$$\begin{array}{r} {\scriptstyle 1\,1} \\ 430 \\ 856 \\ +\quad 27 \\ \hline 1313 \end{array}$$

Provide supervised practice to students who had difficulty with addition sums till they can do it correctly and give homework on these. Both should include numbers requiring complex number facts, zero and numbers with different digits.

Subtraction The following format may be used for developing conceptual understanding of subtraction which may then be generalized to larger numbers.

Teacher	Student
Suppose you have 63 rupees in smallest number of notes. Then how many ten-rupee notes and one-rupee notes do you have.	6 ten- rupee and 3 one-rupee notes.
If you want to buy something costing rupees 27. How many ten-rupee and one-rupee notes you should give?	2 ten-rupee and 7 one-rupee notes.
Suppose the shopkeeper does not have change, what you may do to give him 7 one-rupee notes.	Exchange 1 ten-rupee note for 10 one-rupee notes.
How many ten and one-rupee notes you have now?	5 ten-rupee and 13 one-rupee notes.
After you give the shop keeper 2 ten-rupee and 7 one-rupee notes, how many ten-rupee and one-rupee notes would you have?	3 ten-rupee and 6 one-rupee notes.
How did you obtain it	By subtracting ten-rupee notes from ten-rupee notes and one-rupee notes from one-rupee notes.
Yes, we only subtract the likes as in addition	
How much money is that.	26 rupees.

Subtraction of two numbers may similarly be expressed in expanded notation as follows

$$\begin{array}{lcl} 63 = 6\text{ tens} + 3\text{ units} & = & 5\text{ tens} + 13\text{ units} \\ \underline{-\ 37} = \underline{3\text{ tens} + 7\text{ units}\ } & = & \underline{-3\text{ tens} + 7\text{ units}} \\ & & \ 2\text{ tens} + \ 6\text{ units} = 26 \end{array}$$

or in short from

63
- 37
26

The number 63 from which a number is subtracted is called the **minuend**, 37 the number to be subtracted is called the **subtrahend** and the number 26 obtained by subtraction is called the **difference.**

Provide supervised practice on these to students who had difficulty with renaming in class till they can do it correctly and give home work for practice on their own.

Similarly subtraction of 3 or 4 digits numbers may be carried out by arranging them in columns so that digits in like places are in the same columns and beginning subtraction with units. If the number of units in subtrahend is larger than the units in minuend, rename one unit from tens as 10 units and combine with these units while reducing the number of tens by one and subtract the units. Similarly for tens, hundreds etc. by renaming a unit in higher decade viz., hundreds, thousands respectively and adding it with tens and hundreds, etc. in the number of tens, hundreds, etc. if the subtrahend is larger that that in the minuend.

Illustrate it by many examples using the hierarchy given in table 2.

Subtraction from a number with two or more consecutive zeros may be done in two or more stages e.g.

600= 5H+10T = 5H + 9T +10U
- 253 = -2H + 5T + 3U
= 3H + 4T + 7U = 347

or in short form

600
- 253
347

Multiplication-The following format may be used for developing conceptual understanding of multiplication which may then be generalized to multiplication of larger numbers.

Teacher	Students
Suppose your family is going home for vacations. This cost of one railway ticket is 34 rupees. How much money you need for 6 tickets? While you could add 34 six times it would be more efficient to think of 34 rupees as 3 ten-rupee notes and 4 one-rupee notes for each ticket, and then use multiplication facts.	

How many notes of ten and one-rupee do you need for 1 ticket?	3 ten-rupee and 4 one-rupee notes
How many ten and one-rupee notes do you need for 6 tickets?	18 ten-rupee and. 24 one -rupee notes
If you want to carry the smallest number notes, what you may do?	Exchange 20 one- for 2 ten-rupee notes, would give 20 ten-rupee and 4 one-rupee notes and then exchange 20 ten-rupees notes for two hundred-rupee and 4 one-rupee notes
How many hundred, ten and one-rupee notes do you have?	two hundred-rupee and four one-rupee notes
How much money is that?	204

Similarly we may use expanded notation and renaming for multiplying a two-digit number by a one-digit number

$$\begin{array}{rl} 34 & = 3 \text{ tens} + 4 \text{ units} \\ \underline{\times\ 6} & \underline{\qquad\quad \times 6} \\ & = 18 \text{ tens} + 24 \text{ units} \\ & = 18 \text{ tens} + 2 \text{ tens} + 4 \text{ units} \\ & = 20 \text{ tens} + 4 \text{ units} \\ & = 2 \text{ hundred} + 0 \text{ tens} + 4 \text{ units} \\ & = 204 \end{array}$$

This may be written in short form as

$$\begin{array}{rl} 34 & \\ \underline{\times\ \ 6} & \\ 24 & \rightarrow 6 \times 4 \\ \underline{180} & \rightarrow 6 \times 30 \\ 204 & \end{array}$$

We first multiply 6 by units that gives 24 units which may be renamed as 4 units and 2 tens and written in the column for units, and tens. Then we multiply 3 tens by 6 which gives 18 tens, which is equal to 8 tens and 1 hundred which are written in ten and hundred columns respectively. Zero is written in units' column as there are no units. These two can be added to find 34 × 6 using distributive property of multiplication over addition.

The number to be multiplied is called multiplicand, number by which it is multiplied multiplier, and the number obtained by multiplication the product, e.g. numbers 34, 6 and 204 respectively in the above example.

It may be written in short form as

$$\begin{array}{r} 34 \\ \underline{\times\ \ 6} \\ 204 \end{array}$$

Provide supervised practice on these after assuring mastery of prerequisites for multiplication of a two digit number by a one digit number to students who had difficulty with multiplication and give homework on this. Multiplication may then be extended to the multiplication of a three or more digit numbers by a one digit number in a similar manner.

Teaching of multiplication of a two digit number by a two digit number may be explained by using it and multiplication by tens

$$\begin{aligned} 34 \times 27 &= 34 \times (20 + 7) \\ &= 34 \times 20 + 34 \times 7 \\ &= 680 + 238 \\ &= 918 \end{aligned}$$

or in short form as

$$\begin{array}{rl} 3\ 4 & \\ \times\ \underline{2\ 7} & \\ 238 & \rightarrow 7 \times 34 \\ \underline{680} & \rightarrow 20 \times 34 \\ 918 & \end{array}$$

Multiplication of a three digit, number by a two digit number may be explained by combining the multiplication of a three digit number by a one digit number and tens in a similar manner.

$467 \times 73 = 467 \times (70 + 3) = 467 \times 7 \times 10 + 467 \times 3 = 32690+1401 = 3269 \times 10 + 1401 = 34091$

or in short form as

$$\begin{array}{rl} 467 & \\ \underline{\times\ \ 73} & \\ 1401 & \rightarrow 3 \times 467 \\ \underline{32690} & \rightarrow 70 \times 467 \\ \underline{34091} & \end{array}$$

Multiplication of a three digit number may be explained by combining multiplication of three-digit numbers by one digit number, tens and hundreds as follows:

$463 \times 375 = 463 \times (300 + 70 + 5) = 463 \times 300 + 463 \times 70 + 463 \times 5$

$= 463 \times 3 \times 100 + 463 \times 7 \times 10 + 463 \times 5$

$= 1389 \times 100 + 3241 \times 10 + 2315 = 138900 + 32410 + 2315 = 173625$

or in short form as

$$\begin{array}{rl} 463 & \\ \underline{\times\ 375} & \\ 2315 & \rightarrow 5 \times 463 \end{array}$$

32410→70 × 463
$\underline{138900}$→100 × 463
173625

Provide supervised practice on multiplication to students who had difficulty with multiplication using the hierarchy of skills in Table 3 and give homework referenced to these.

Division - The following formats may be used for teaching conceptual understanding of division by one digit numbers which may then be generalized.

Division of two-digit number with a remainder

Teacher	Students
Suppose you have 23 rupees and you want to divide it equally among 7 friends. How much money can you give to each.	3 rupees
How much money is left?	2 rupees

We can write it in short form as

$$\begin{array}{r} 3 \\ 7\overline{)23} \\ \underline{21} \\ 2 \end{array}$$

The number to be divided-23 is called the **dividend**, number of groups in which it is to be equally divided -7 the **divisor** the numbers which gives the share of each group-6 the **quotient** and the left over part 2 as the **remainder**

Division of a two digit number by a one digit number with two digit quotients and a remainder

Teacher	Students
Suppose you have 5 ten rupees notes and want to divide it equally among 4 sisters and brothers can you give a ten rupee note to each. If yes, how many?	Yes one ten-rupee note to each
How many ten rupee notes have been used?	4 × 1=4 ten-rupee notes
How many remain?	One ten-rupee note
What you may do to divide it rupee	Exchange it for ten one-notes
Then how many one rupee notes you can give to each	2 one-rupee notes
How many one-rupee notes have been distributed	8 one- rupee notes
How much money is left?	2 one- rupee notes
How much money did each get?	One ten- rupee notes and two one- rupee notes

How much money is that 12 rupees

We may write it in symbols as

12
4√50
4
10
8
2

Note tens in the quotient are written above tens and ones above ones. Similarly to divide a three digit number by a one digit number e.g. $6\overline{)778}$, we first look at hundreds, if their number is larger than divisor, we divide the hundreds first, change the left over hundreds to tens and combine that with tens, divide these and rename the left over tens to ones and add and divide those e.g.

129
6√778
6
17
12
58
54
4

Division of a two digit number by a two digit number with one digit quotient $33\overline{)78}$. It may be conducted by making a table of multiples of divisor till the multiple is less than the dividend e.g. 33 × 1 = 33, 33 × 2 = 66, 33 × 3 = 99. As 33 × 3=99, exceeds the dividend, 2 is the desired quotient and remainder may be obtained by subtracting the product of quotient and divisor from the dividend Thus.

2
√78
66
12

This involves too much work if the dividend is large. We may estimate it by rounding off the two digit numbers to the nearest ten - the same as the digit in tens place if the unit digit is less than 5 and one more than the digit in tens place if the digit in unit place is 5 or more. For example 33, 78, 45 rounded to nearest ten are 3, 8 and 5 tens or 30, 80 and 50 respectively. We find the trial quotient by dividing 8 by 3, that is, 2 as quotient. This may not always give a correct quotient. Sometimes it may give an underestimate; in that case the remainder would be larger than the divisor you would need to consider a quotient one more than the trial quotient. It may also give you an overestimate in which case the product of trial quotient and divisor would be larger than tens in the dividend. You would need to take a number one less than the trial quotient and check again.

Division of a three digit number by a two digit number with a two digit quotient

$41\overline{)867}$

If the first two digits are greater than the divisor, estimate the digit in tens place in the quotient by rounding off the first two digits to the nearest ten that is 9 and also the divisor that is 4 and divide them to estimate the digit in tens place. The trial quotient would be 2.

Multiply the divisor by tens digit in the quotient that is 82 and as the product is less than number of tens in the dividend.

Subtract the product from the dividend that gives 4 tens.

Rename the left over tens as ones and combine with the ones, we have 47 ones or 5 tens, dividing by 4 tens, we get 1as trial quotient. Write 1 in units place in the quotient, multiply it with the divisor that give 41 that is less than 47, subtract it from 47, we get 6 as remainder. We need to check the remainder at every stage also, which should be less than the divisor. If not, then increase the quotient by 1.

$\underline{120}$
41√867
$\underline{82}$
47
$\underline{41}$
6

Give more examples and provide practice on division at each level of the hierarchy given in Table 5 after ensuring the mastery of prerequisites and easier skills.

IV-CONCLUDING COMMENTS

We have tried to discuss remedial teaching with minimal demands in view of paucity of resources. However, there is still a need for some support by Institutes of Advanced studies in Education and schools. Institute of Advanced studies in Education may motivate teachers for remedial teaching in in-service programmes and train them for it, develop aids for remedial teaching such as teaching guides and teaching learning materials which should include exercises referenced to established types of difficulties, mastery tests, diagnostic tests and achievement tests. The schools should organize these and procure parental support and cooperation for it.

We have discussed diagnosis of weaknesses and remedial teaching in a limited context viz. at the beginning of upper primary stage in class VI in the context of government schools in Delhi. But, it should not begin and end here.

There is a need to assure mastery after teaching of minimum essentials for further learning and before teaching of topics for which these are essential prerequisites and doing remedial teaching if necessary at all stages of education. As algebra is introduced in class VI, there is a need to provide for and assure mastery of its basic concepts and skills and ensure it before teaching algebra in class VII. Similarly before teaching of rational numbers, mastery of prerequisite concepts in fractions, integers needs to be ensured by testing for mastery and undertaking remedial teaching if necessary.

We have mainly confined to pedagogical causes and weaknesses of the group only. It is desirable to consider causes of individual difficulties including other causes such as physical disability, emotional stress, problems at home, low aptitude and take corrective measures for those or at least keep them in view while teaching and setting standards.

Efforts should be made to improve the curricula in primary schools to prevent the need for remedial teaching by providing for mastery of minimum essential and retention of those by taking the measures given below:

- Selecting objectives consonant with developmental stage, prior learning experiences and interests of the students.
- Sequencing objectives so that prerequisites and easier skills are taught first and complex skills are presented using small increments after they have mastery of easier skills.
- Teaching concepts and procedures with understanding for longer retention, applications and further learning by using manipulative first, then pictorial aids, written and oral symbols or words, and situations in daily life.

- Providing massed practice for symbols, facts and skills and providing opportunities for distributed practice later for retention.
- Providing for individual differences as children progress at different rates, have different types of difficulties and need different amount of practice for attaining mastery.
- Evaluating continuously formally and informally for monitoring mastery and guiding teaching and learning.
- Examining types of errors for wrong answers while evaluating corrective measures.

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APPENDIX 1[2]

Performance of Students on Numeration System

Objective	Item	No. of students responding to the item	Proportion of students responding correctly
Write a number given in figures in words	Write the following numbers in words:		
	189	134	0.58
	7059	120	0.38
	4008	120	0.51
	54679	127	0.32
	700005	126	0.41
Translate a number given in words in figures	Write the following numbers in figures:		
	Seven hundred six	126	0.41
	Five thousand three hundred eight	127	0.34
	Fifty three thousand fifty three	120	0.30
	Six lakh seventy two thousand four hundred twenty three	120	0.44
	Eight lakh seventeen	134	0.30
Compare Numbers	Compare numbers given below by writing		

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[2] Institute of Advanced studies in Education in collaboration with Maulana Azad Centre for Elementary and Social of Education Delhi has adopted Jahangirpur Block of government schools of Delhi Directorate of Education for school development. Jahangirpuri is a resettlement locality situated in North Delhi in the vicinity of Azadpur Market. Many schools in the area are in a low lying area which gets water logged in rainy season making teaching difficult. All secondary schools are under Directorate of Education Delhi. They all follow NCERT text books, have trained and qualified teachers with mathematics as a subject at least up to graduate level, use Hindi as medium of instruction, have large classes and start with class VI. The performance of these students in board examinations in class X and XII is low. A crucial problem identified by mathematics teachers in improving performance in mathematics in these schools is the lack of fundamentals in mathematics. A study was undertaken in 1995 by Institute of Advanced studies in Education in collaboration with me to identify areas in which remedial teaching should be undertaken to improve learning in class VI. It was mainly restricted to adequate mastery of whole numbers as learning in class VI mainly depends on that.

	>, <, or =	127	0.54
	56......65	130	0.75
	430.......403		
Understands the concept of place value	What number comes just after 999	120	0.27
	What number comes just before 10,000	134	0.20
	In 4035 name the place of digit		
	5	120	0.12
	3	120	0.27
	0	134	0.30
	4	126	0.30
	In 6723 name the digit		
	in	120	0.27
	units place	134	0.58
	tens place	126	0.28
	hundreds place	127	0.32
	thousands place	120	0.15
	Write the smallest number having digits 1, 8, 5, 4 using each digit once only	127	0.06
	Write largest number having digits 2, 6, 7, 0 using each digit once only		

Performance of Students on Addition

Objective	Item	Number of students	Proportion students responding to the item correctly
Understand the concept of addition	A brother is 5 years older than his brother If the age of younger brother is 8 years. What is the age of older	126	0.06

	brother?		
	Write the next number of the number series given below		
	1, 3, 5, 7 ___	127	0.35
	6, 12, 18, 24. ___	120	0.50
Execute addition	Simplify		
	325 + 441	138	0.99
	783 + 104	126	0.80
	385 26 + 122	127	0.90
	3856 2488 1432 + 1678	120	0.83
	3425 16 170 + 8	120	0.73
Apply addition to solve problems in everyday life	A farmer had Rs.500 he had to borrow Rs 300 to buy seeds. How much money did he spend on seeds?	127	0.38
	A train starts its journey at 4'0 clock in the evening from Delhi. It takes 3 hours 50 minutes to reach Ambala. At what time would it reach Ambala?	127	0.35
	Rita bought a book	120	0.43

	costing Rs.14.50, a copy costing 3 rupees and a pencil costing 70 paise. how much money she would have to pay to the shopkeeper?		
	How many millilitres are there in one litre	138	0.23
	A sweet maker bought 45 litres of milk. He used 15 litres and 600 millilitres for making one type of sweet and 20 litres 200 millilitres for making another type of sweet. How much milk did he use up in making sweets.	138	0.03

Performance of Students on Subtraction

Objective	Item	Number of students responding to the item	Proportion of students responding correctly
Understand concept of subtraction	Write next number of the number series 18,15,12, 9...	120	0.45
	Verify the answer of subtraction given below by using addition 732-258=474	120	0.05
Execute subtraction	Simplify 352 - 187	126	0.48
	724 - 578	120	0.62
	503 - 87	127	0.46

	700 - 86	124	0.57
	8000 -4264	134	0.55
Apply subtraction to solve problems in everyday life	Sanjay bought a book for Rs.23. If he gave a fifty rupees note to the shopkeeper, how much money should the shopkeeper return?	120	0.62
	Raju had Rs. 25, his father gave him some money, then he had. Rs 70. How much money did his father give him.	134	0.59
	Rajdhani Express starts at 4.05 in the evening from Delhi and reaches Bombay the next morning at 8.50. How much time does the journey from Delhi to Bombay take?	126	0.07

Performance of Students on Multiplication

Objective	Item	Number of students responding to the item	Number of students responding correctly
Understand the concept of multiple	Which of the following numbers is not a multiple of 7 7, 14, 21, 28, 42, 49, 54, 63	120	0.39
	Write the next number of the number series 3, 9, 27, 81, ___	134	0.42
Execute multiplication	Multiply		
	7 × 100	134	0.77
	179 × 0	120	0.44
	30 × 40	127	0.51
	$\begin{array}{r} 172 \\ \times\ \underline{23} \end{array}$	126	0.53
	400 × 800	126	0.28
	478 × 900	120	0.39
	$\begin{array}{r} 382 \\ \times\ \underline{546} \end{array}$	127	0.32
	$\begin{array}{r} 523 \\ \times\ \underline{807} \end{array}$	120	0.35
Properties of multiplication	Is 13×14 =14×13?	120	0.42
Apply multiplication To solve problems in everyday life	A woman cooks 400gm of rice per day. How much rice does she need for a week?	134	0.56
	A train covers 40 km., in one hour. How much distance would it cover in 6 hours	134	0.68
	Vivek bought 4 copies at the rate of Rs. 3	134	0.22

	per copy and a book for Rs. 15. How much money he should pay to the shopkeeper?		
	Rajan bought two kg. mangoes selling at Rs.12 per kg. He gave a fifty rupee note to the shop-keeper. How much money he should return to Rajan?	120	0.16
Convert units	How many centimetres are there in one metre.	120	0.36
	How many centimetres are there in 2 metres and 25 centimetres.	126	0.02
	How many grams are there in one kilogram?	120	0.39
	How many grams are there in 3 kilograms and 10 grams?	120	0.03
	How many millimetres are there in one litre?	126	0.23
	How many millilitres are there in 2 litres and 135 millilitres?	120	0.01

Performance of Students on Division

Objective	Item	Number of students	Proportion of students responding correctly
Understands concept of division	Write next number of the series 32, 16, 8, 4, ___	126	0.40
Executes Division	Find 5)0	134	0.25
	179)179	126	0.56
	9)274	127	0.24
	23)94	120	0.42
	47)2634	126	0.13
	67)1325	120	0.08
	36)3605	134	0.22
Applies division to problems in every day life	A train travels 180 km. in 5 hours. How much distance does it cover in one hour?	120	0.39
	Two metres of cloth are required for making a shirt. How many shirts can be made in 18 metres of cloth?	120	0.50
	You can buy three dollars in one hundred rupees. About how much is the cost of a dollar in rupees?	120	0.39
	How many ribbons of length 25 cm., can be made from a one metre long ribbon?	127	0.37

Integrates multiplication and division	Simplify		
	$\frac{9 \times 8}{4}$	127	0.24
	$\frac{100 \times 5 \times 3}{100}$	126	0.20
	$\frac{8000 \times 7 \times 2}{100}$	127	0.23
Find a specific percentage of number	Find		
	5% of 90	120	0.01
	7% of 1000	120	0.21
	8% of 2500	134	0.03
Applies both multiplication and division to solve problems in everyday life	The cost of 100 gm. of ginger is Rs.4, what is the cost of 250 gm of ginger?	127	0.42
	Eggs cost Rs. 18 per dozen. What is the cost of 8 eggs?	120	0.02

APPENDIX 2

Addition and Subtraction fact families

2 + 2 = 4	4 – 2 = 2		
2 + 3 = 5	3 +2 = 5	5 – 2 = 3	5 – 3 =2
2 + 4 = 6	4 + 2 = 6	6 – 2 = 4	6 – 4 = 2
2 + 5 = 7	5 + 2 = 7	7 – 2 = 5	7 – 5 = 2
2 + 6 = 8	6 + 2 = 8	8 – 2 = 6	8 – 6 = 2
2 + 7 = 9	7 + 2 = 9	9 – 2 = 7	9 – 7 = 2
2 + 8 = 10	8 + 2 = 10	10 – 2 = 8	10 –8 = 2
2 + 9 = 11	9 + 2 = 11	11 – 2 = 9	11 – 9 = 2
3 + 3 = 6	6 – 3 = 3		
3 + 4 = 7	4 + 3 = 7	7 – 3 = 4	7 – 4 = 3
3 + 5 = 8	5 + 3 = 8	8 – 3 = 5	8 – 5 = 3
3 + 6 = 9	6 + 3 = 9	9 – 3 = 6	9 – 6 = 3
3 + 7 = 10	7 + 3 = 10	10 – 3 = 7	10 – 7 = 3
3 + 8 = 11	8 + 3 = 11	11 – 3 = 8	11 – 8 = 3
3 + 9 = 12	9 + 3 = 12	12 – 3 = 9	12 – 9 = 3
4 + 4 = 8	8 –4 = 4		
4 + 5 = 9	5 + 4 = 9	9 – 4 = 5	9 – 5 = 4
4 + 6 = 10	6 + 4 = 10	10 – 4 = 6	10 – 6 = 4
4 + 7 = 11	7 + 4 = 11	11 – 4 = 7	11 – 7 = 4
4 + 8 = 12	8 + 4 = 12	12 – 4 = 8	12 – 8 = 4
4 + 9 = 13	9 + 4 = 13	13 – 4 = 9	13 – 9 =4
5 + 5 = 10	10 – 5 = 5		
5 + 6 = 11	6 + 5 = 11	11 – 5 = 6	11 –6 = 5
5 + 7 = 12	7 + 5 = 12	12 – 5 = 7	12 – 7 = 5
5 + 8 = 13	8 + 5 = 13	13 – 5 = 8	13 – 8 = 5
5 + 9 = 14	9 + 5 = 14	14 – 5 = 9	14 – 9 = 5
6 + 6 = 12	12 – 6 = 6		
6 + 7 = 13	7 + 6 = 13	13 – 6 = 7	13 – 7 = 6
6 + 8 = 14	8 + 6 = 14	14 – 6 = 8	14 – 8 = 6
6 + 9 = 15	9 + 6 = 15	15 – 6 = 9	15 – 9 = 6
7 + 7 = 14	14 – 7 = 7		
7 + 8 = 15	8 + 7 = 15	15 – 7 = 8	15 – 8 = 7
7 + 9 = 16	9 + 7 = 16	16 – 7 = 9	16 – 9 = 7
8 + 8 = 16	16 – 8 = 8		
8 + 9 = 17	9 + 8 = 17	17 – 8 = 9	17 – 9 =8
9 + 9 = 18	18 –9 = 9		

APPENDIX 3

Multiplication and Division fact families

2×2 = 4	4 ÷ 2 = 2		
2 × 3 = 6	3 × 2 = 6	6 ÷ 2 = 3	6 ÷ 3 = 2
2 × 4 = 8	4× 2 = 8	8 ÷ 2 = 4	8 ÷ 4 = 2
2 × 5 = 10	5 × 2 = 10	10 ÷ 2 = 5	10 ÷ 5 = 2
2 × 6 = 12	6 × 2 = 12	12 ÷ 2 = 6	12 ÷ 6 = 2
2 × 7 = 14	7 × 2 = 14	14 ÷ 2 = 7	14 ÷ 7 = 2
2 × 8 = 16	8 × 2 = 16	16 ÷ 2 = 8	16 ÷ 8 = 2
2 × 9 = 18	9 × 2 = 18	18 ÷ 2 = 9	18 ÷ 9 = 2
3 × 3 = 9	9 ÷ 3 = 3		
3 × 4 = 12	4 × 3 = 12	12 ÷ 3 = 4	12 ÷ 4 = 3
3 × 5 = 15	5 × 3 = 15	15 ÷ 3 = 5	15 ÷5 = 3
3 × 6 = 18	6 × 3 = 18	18 ÷ 3 = 6	18 ÷ 6 = 3
3 × 7 = 21	7 × 3 = 21	21 ÷ 3 = 7	21 ÷ 7 = 3
3 × 8 = 24	8 × 3 = 24	24 ÷ 3 = 8	24 ÷ 8 = 3
3 × 9 = 27	9 × 3 =27	27 ÷ 3 = 9	27 ÷ 9 = 3
4 × 4 = 16	16 ÷ 4 = 4		
4 × 5 = 20	5 × 4 = 20	20 ÷ 4 = 5	20 ÷ 5 = 4
4 × 6 = 24	6 × 4 = 24	24 ÷ 4 = 6	24 ÷ 6 = 4
4 × 7 = 28	7 × 4 = 28	28 ÷ 4 = 7	28 ÷ 7 = 4
4 × 8 = 32	8 × 4= 32	32 ÷ 4 = 8	32 ÷ 8 = 4
4 × 9 = 36	9 × 4 = 36	36 ÷ 4 = 9	32 ÷ 9 = 4
5 × 5 = 25	25 ÷ 5 = 5		
5 × 6 = 30	6 × 5 = 30	30 ÷ 5 = 6	30 ÷ 6 = 5
5 × 7 = 35	7 × 5 = 35	35 ÷ 5 = 7	35 ÷ 7 = 5
5 × 8 = 40	8 × 5 = 40	40 ÷ 5 = 8	40 ÷ 8 = 5
5 × 9 = 45	9 × 5 = 45	45 ÷ 5 = 9	45 ÷ 9 = 5
6 × 6 = 36	36 ÷ 6 = 6		
6 × 7 = 42	7 × 6 = 42	42 ÷ 6 = 7	42 ÷ 7 = 6
6 × 8 = 48	8 × 6 = 48	48 ÷ 6 = 8	42 ÷ 8 = 6
6 × 9 = 54	9 × 6 =54	54 ÷ 6 = 9	54 ÷ 9 = 6
7 × 7 = 49	49 ´÷ 7 = 7		
7 × 8 = 56	8 × 7 = 56	56 ÷ 7 = 8	56 ÷ 8 = 7
7 × 9 = 63	9 × 7 = 63	63 ÷ 7 = 9	63 ÷ 9 = 7
8 × 8 = 64	64 ÷ 8 = 8		
8 × 9 = 72	9 × 8 = 72	72 ÷ 8 = 9	72 ÷ 9 = 8
9 × 9 = 81	81 ÷ 9 = 9		